**2.2 Derivation of the Transfer Function:**

The Transfer Function *G(s)* is produced using the formula:

For which Y(s) is the output and X(s) is the input of the system. For this case, the system dynamics is depicted by the following differential equations:

Within the system, the following symbols, , , and denote pitch angle, angle of attack, pitch rate and the deflection angle of the elevators respectively. Which harbour crucial roles in regulating the pitch control of the aircraft. To obtain the transfer function of the system, the Laplace transform method is applied on each of the differential equations in which zero initial conditions are assumed. The transfer function will utilise the aforementioned results and take the form of:

The Laplace transformations of each of the differential equations are as follows:

The final Transfer Function after further algebraic manipulation to find in terms of is:

Where ), pitch angle, represents the output and , deflection angle of the elevators, represents the input.

The final form of the Transfer Function is the concise representation that encompasses the dynamics of the system. By representing the relationship between the system’s input and output signals in the frequency domain. Hence serving as a mathematical model exhibiting the essential characteristics of the system’s behaviour. This Transfer Function will be the used as the basis for the controller design that will be further investigated as this report progresses.

**2.3 Preliminary Analysis**

To illustrate the performance of the aircraft system itself, analysis will be carried out to study the properties of the open-loop dynamics. For this to be achieved, poles and zeros, stability and system responses will be examined.

**2.3.1 Stability Analysis**

This subsection focuses on the poles and zeroes of the Transfer Function. Upon examination of *G(s)*, it is observed that there is one zero located at the origin, this being s=0, while there are two poles located within the roots of the quadratic polynomial on the denominator of the Transfer Function at, These poles, Q, are the poles of F. This zero being at the origin connotes that the output of the system,), is directly proportional to its input, with no additional dynamics.

**2.3.2 Pole Location Analysis**

The next aspect to be examined with regards to stability is the poles position in the complex plane. For the Transfer Function calculated, the poles are located at = -0.368+0.32753015j and =-0.368+0.32753015j. These poles having negative real parts indicates stability, as the poles are in the left-half of the complex plane exhibiting the system’s natural reaction to lean towards its equilibrium state over time. Indicating that in instances in which the system would experience external disturbances, in this case, turbulence, wind gusts and structural flexibility equilibrium restoration occurs. Hence, the negative real part suggests that the response diminishes exponentially over time, approaching zero. Exhibiting stability within the system, as the disturbances affecting its performance would lessen within time.

**2.3.3 System Response Analysis**

**Bode Plot**

**A graph of a function

Description automatically generated**

Figure 1. Bode plot measuring Magnitude (dB) against Frequency (rad/sec) and Phase (deg.) against Frequency (rad/sec). The Bode plot exhibits how the output signal (pitch angle) changes with frequency. To show the gain as a function of frequency. The phase plot, displays the phase shift initiated by the system at different frequencies, indicating the phase delay or advance of the output signal relative to the input signal as a function of frequency.

The Bode Plot shows the gain at different frequencies, as steady decline in magnitude is seen at -20 dB, showing the system acting as a single-order low-pass filter, which attenuates higher frequencies at the said rate of -20 dB. The corner frequency, the point of magnitude roll off, is determined to be at 0.1rad/sec, from the point at which the line intersects the 0 dB axis. The phase plot section displays a steadily decreasing phase lag starting from 0 degrees at low frequencies, then approaching -90 degrees as the frequency increases, a common characteristic in single-order systems. Additionally, the phase lag at the corner frequency is approximately -45 degrees. Indicating that the system’s response becomes more synchronised with the input signal as the frequency increases, this shows its rapid response to changes in control inputs or external disturbances at these higher frequencies. Especially with the phase margin estimated to be at 90 degrees, showing a stable system. The low-pass filter and decreasing phase lag characteristics combined shows the system’s frequency-dependent behaviour, which can also be modelled as a first-order system.

**A graph with a line

Description automatically generatedStep response**

Figure 2. A graph of the step response against time (sec) of the Transfer Function of the control system. The step response is the time-domain reaction of the system to sudden instantaneous changes in the input. In this case a step input. Providing insight into the information about the system’s transient behaviour and performance characteristics.

From the observations made with regards to the step response, the rise time at 2.5 seconds shows the quick system responds to the step input. Implying a responsive system. There is also no presence of an overshoot, suggesting a well damped system. Achieving stability with no oscillations ot excessive deviation from the aircraft’s trajectory. Around 7 seconds there is a settling time which indicates how long it would take for the system’s response to reach and remain within the steady-state value. This value being around 3.7 in the response scale. Representing the desired equilibrium point for the control system. By reaching this value, it ensures that the aircraft will maintain its intended flight parameters, such as altitude or pitch angle under the control system’s normal operating criteria.

**A graph with a line

Description automatically generated Impulse Response**

Figure 3. The impulse response against time (sec) showing the system’s reaction upon undergoing a brief impulse or spike input. Which are sudden disturbances, this aids in assessing the system’s stability and responsiveness.

The results seen from the impulse response has a peak value around 0.82, illustrating the maximum amplitude of the curve. This indicates the magnitude of the system’s initial response to the impulse input. The peak time at which this value occurs at is 2.5 seconds, this provides insight into the responsiveness of the system. The settling time is found from 2.5 seconds to 10 seconds, giving a 7.5 second interval for the system to return to its equilibrium conditions, showing its fast stabilisation and quick recovery from transient disturbances. The rapid decline in oscillations after the initial peak would suggest that the system is well-damped, implying that the system’s transient response quickly settles without any excessive oscillations or instability.